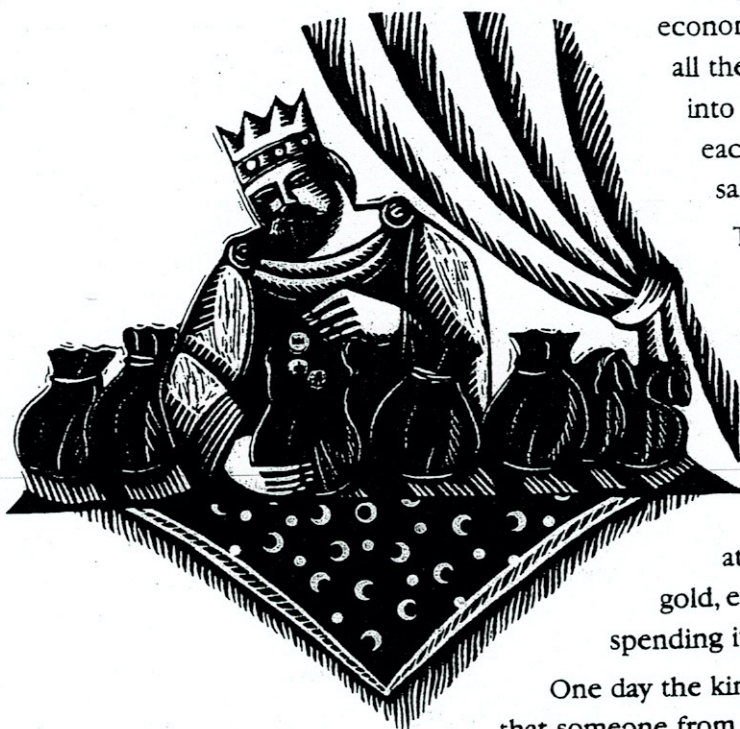


Eight Bags of Gold



Once upon a time there was a very economical king who gathered up all the gold in his land and put it into eight bags. He made sure that each bag weighed exactly the same amount.

The king then chose the eight people in his country whom he trusted the most, and gave a bag of gold to each of them to keep safe for him. On special occasions he asked them to bring the bags back so he could look at them. (He liked looking at his gold, even though he didn't like spending it.)

One day the king heard from a foreign trader that someone from the king's country had given the trader some gold in exchange for some merchandise. The trader couldn't describe the person who had given her the gold, but she knew that it was someone from the king's country. Since the king owned all of the gold in his country, it was obvious that one of the eight people he trusted was cheating him.

The only scale in the country was a pan balance. This scale wouldn't tell how much something weighed, but it could compare two things and indicate which was heavier and which was lighter. The person whose bag was lighter than the others would clearly be the cheat. So the king asked the eight trusted people to bring their bags of gold to him. Being very economical, the king wanted to use the pan balance as few times as possible. He thought he might have to use it three times in order to be sure which bag was lighter than the rest. His court mathematician thought that it could be done in fewer weighings. What do you think?

To answer this question, follow these steps.

1. Develop a scheme for comparing bags that will always find the light one.
2. Explain how you can be sure that your scheme will always work.
3. Explain how you know that there is no scheme with fewer weighings that will work.

Note: Each comparison counts as a new weighing, even if some of the bags are the same as on the previous comparison.