What to Do

Listen and guess.

Your teacher will play a recording of the first and second harmonics. Listen to the difference in pitch and look at their vibration patterns on the Standing-Wave Vibrations worksheet. Discuss what you hear and see, and make a logical guess as to what the frequency of the second harmonic is in relation to the first.

Hints

- Higher-sounding notes vibrate with higher frequencies.
- The node of the second harmonic is exactly in the middle of the string.

Calculate harmonic frequencies.

Using the pattern you observed, fill in on your worksheet the frequencies for the rest of the harmonics.

Find the harmonic frequencies on the piano.

Listen to the examples played by your teacher and place an X directly below the piano key that sounds like the same note as the harmonic you hear. Repeat this process until you have located all of the harmonics on the piano keyboard. Below each X on the diagram, write the general frequency value for each harmonic in terms of the fundamental, *f*.

Understand musical intervals.

Musicians call the distance between two notes an *interval* and have special names for them. Use the chart below to find the names of the intervals between each pair of harmonics. Write these names in the table on your worksheet.

Interval name	Keys on piano
Second	spans 2 white keys
Third	spans 3 white keys
Fourth	spans 4 white keys
Fifth	spans 5 white keys

Find the ratio for the interval.

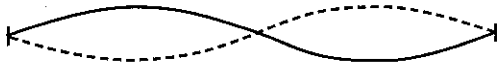
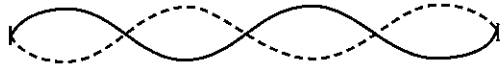
Calculate the frequency ratio between the top note and the bottom note of each interval and place it on the table on your worksheet.

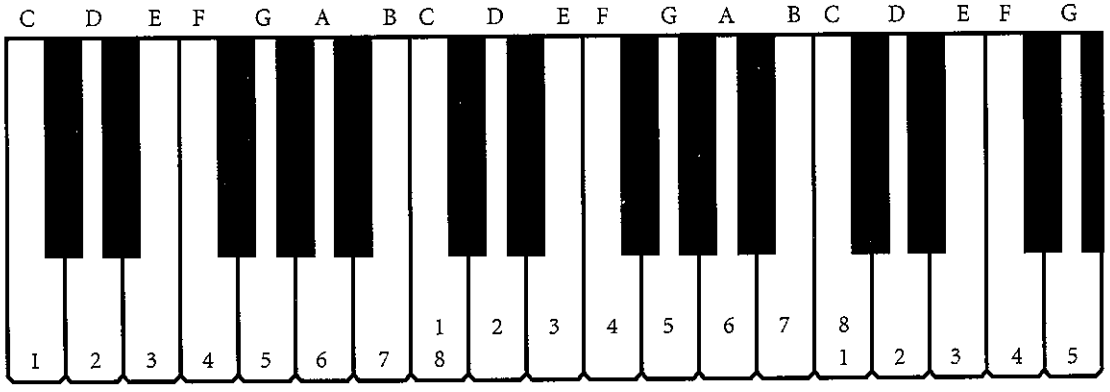
Create the major scale.

Follow the steps Pythagoras followed as you complete the Notes As Vibrations worksheet and find the frequency ratio of each note to the fundamental.

Name

STANDING-WAVE VIBRATIONS

	Frequency		Vibrating string pattern
	Example	General	
First harmonic	130	f	
Second harmonic			
Third harmonic			
Fourth harmonic			
Fifth harmonic			



Harmonic frequency	
Musical interval	
Ratio for interval	

Name

Notes As Vibrations

You have just learned how the natural vibrating frequencies occur in the musical scale used by modern Western music. The exploration you are about to make develops the major scale. It was followed by Pythagoras about 2,500 years ago.

1. Refer to the diagram of the piano keyboard. If you were to lay intervals end to end (with the last note of one overlapping the first note of the next), what two intervals of harmonic tones would fit exactly within one octave?

2. Now consider the frequency ratios for these two intervals and write them here.

3. An octave has a ratio of $\frac{2}{1}$. What mathematical operation (addition, subtraction, multiplication, division) between the two intervals you found in the first step and whose interval ratios you listed in the second step gives $\frac{2}{1}$? Experiment. Show your work.

4. Your task is to find the frequency ratios of the other four notes of the major scale relative to the first tone. Use the operation you just discovered. Like Pythagoras, you may use only ratios of the fifth and the octave. You do not need to calculate the intervals for the fourth and fifth. Pythagoras used the natural harmonics for these intervals. Show your work in the spaces provided in the table on the following page.

Hints

- Start with tone number 1. Experiment to see if you can move up the piano by some number of fifths or octaves and then down by another number of fifths or octaves to land on one of the tones you are trying to find. If you find a combination that works, you can use operations with the frequency ratios to find the frequency ratio for the final note.
- To obtain the frequency of a note a fifth higher than a given note, multiply the ratio for that note by $\frac{3}{2}$.

Notes As Vibrations (continued)

- To obtain the frequency of a note a fifth lower than a given note, divide the ratio for that note by $\frac{3}{2}$.
- The same process works for the octave: Multiply by 2 to find the frequency of a note an octave higher, and divide by 2 to find the frequency of a note an octave lower.

Musical interval	Frequency ratio to fundamental
The fundamental	1
The second	
The third	
The fourth	$\frac{4}{3}$
The fifth	$\frac{3}{2}$
The sixth	
The seventh	