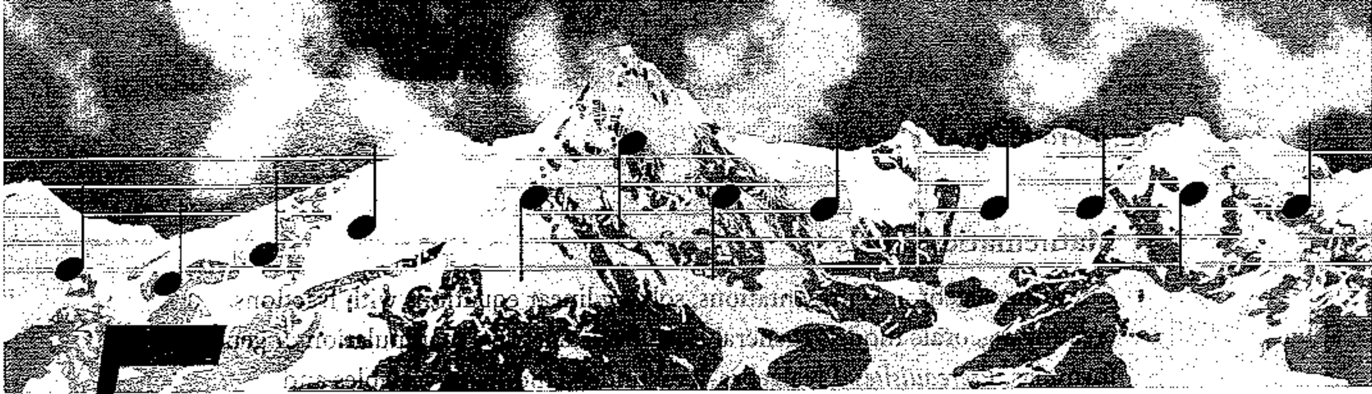


5

Record-Producer Algebra

Using Algebra to Perform Rap Music

TRACKS 29–34

In Record-Producer Algebra, students play the role of a record producer working with a singer, a songwriter, and a band to record a segment of rap music. Producers are responsible for all aspects of the recording process, including writing arrangements or programming computer-sequenced parts that must adhere to requirements set by the artist. Using information about a rap segment, students calculate entrance points and phrase lengths for various vocal parts that have been designated to end at specific places in the musical recording. Students explore guess-and-check strategies and arithmetic solutions of their own design, and they perform vocal renditions of their answers for each problem. Your coaching helps students see algebraic strategies that assign variables, create equations, and generalize all parameters. The general formulas can be used to alter the existing vocal parts and create new arrangements.

Through this activity, students see how arithmetic solutions that they devise and confidently apply are actually the steps in solving an algebraic equation. They are doing algebra without realizing it. Students are not given a strategy. Instead, they obtain numerical information to solve the problems by listening to the CD tracks and counting the beats. Students express their strategies in words as a transition to algebraic solutions. As the problems become more difficult, students see the usefulness of an algebraic equation as a tool.

Mathematics topics

Counting, algebraic representations, solving linear equations with fractions, creating algebraic models (general formulas), algebraic manipulation of general formulas. *Prerequisites:* Prealgebra-level understanding of variables and equations.

Music topics

Beats, time, measures, pop/rock song arranging, vocal phrasing, rap vocal performance. *No prerequisites.*

Use with the primary curriculum

- During the study of algebraic word problems
  Record-Producer Algebra can be a valuable enrichment activity during units dealing with the translation of real-world scenarios into algebraic models.
- As enrichment for algebraic symbol-manipulation techniques for solving equations
  This activity provides an interesting application of equations and gives practice in solving them.
- Between units in any mathematics course
  Developing algebraic skill and understanding is an ongoing process that can be practiced in any mathematics course. This activity will enrich algebra skills through a different kind of learning experience. Variety in instruction can keep all students interested and involved.

Objectives

- To expand awareness of the use of algebra
- To foster understanding of the connection between arithmetic solutions and algebra
- To use algebraic formulas to solve real-world problems
- To provide opportunities for practicing algebraic manipulations
- To engage students in a fun, unique classroom experience

Student handouts

- Algebra: Everyone's Doing It (reading; one per student)
- The Producer's Problems (resource page; one per group)
- The Producer's Workstation (worksheet; one per student)
- Music/Rap Charts (worksheet—optional; two per group)

Materials

- CD tracks 29–34
- Blank music manuscript paper (optional)
- Overhead transparency of Music/Rap Charts

Instructional time

40–80 minutes

Instructional format

Because students are creating algebraic representations from their own arithmetic solutions, the exercises follow a constructivist format that helps students build on their own understanding. Students create strategies, make connections, and solve problems with minimal teacher intervention. The students' work is then mined by you for algebraic content, and you reveal this content to the students to inform their subsequent work. The activity alternates between students working in pairs or small groups and the whole class coming together to listen to the CD track. The group sessions of listening and discussing limit students' freedom to work at their own pace, because all students must complete the same problem within a short time. In a sense, the classroom flow oscillates between whole-class, group experience (the norm for a band class) and independent problem solving.

It can be fun to assign students the task of creating their own lyrics for Problems 1, 2, and 4 before class. If you choose to do this, you will have to specify the number of syllables in order for each problem to work as written mathematically. (Problem 1 needs seven syllables, Problem 2 needs three syllables, and Problem 4 needs two syllables.) Though this can mean more work, you may have one student (linguistically gifted but mathematically challenged) who would love to have the opportunity to contribute with confidence in mathematics class.

TRACKS 31–34

The music CD includes tracks that demonstrate how the rap problems sound when they are performed correctly. You may want to listen to them as you prepare to teach the activity, but don't play them for the class unless your students are having trouble and need to hear a sample. Track 31 demonstrates a correct performance of Problem 1; track 32 is for Problem 2; track 33 is for Problem 3; and track 34 is for Problem 4.

Student preparation

Many students need some time to warm up psychologically to the idea of singing and counting beats in a mathematics class. Give them the reading, Algebra: Everyone's Doing It, a day before the activity, and spend five minutes discussing the ideas it presents. If this is not possible, at least give students some idea of what you will be doing on activity day. The following discussion point gives more information about Leibniz, who is mentioned in the reading.

Your students may be interested in learning that Gottfried Wilhelm Leibniz, a scholar of law and philosophy and a career diplomat, used his spare time to explore mathematics. His studies of infinite series led him to the basic ideas of calculus. Sir Isaac Newton had discovered these ideas a few years earlier, but both men developed the ideas independently. Leibniz was the first to publish the ideas of differential calculus in 1684.

ACTIVITY SCRIPT

STEP 1 Set the context and establish the concept of beat

TRACK 29

Ask students to read the opening paragraph on the resource page to establish the context. Solving the producer's problems will require students to identify the fundamental element of rhythm in music: the beat.

Refer to the reading and discuss how musicians count beats. Solicit explanations and experiences from musicians in the class. Ultimately, you need to establish that the beat is the fundamental pulse used to measure where all of the events in a piece of music occur. Play CD track 29 to demonstrate where the beat falls in the rhythm that students will rap to.

The CD track marks only the location of the beats; it does not even imply assigning numbers to the beats. Establishing the need to count beats should come from students as an outgrowth of their need to solve the problems.

Musical students in your class may comment that beats are counted and organized into measures. Measures are a means by which beats are grouped to help organize the structure of music as well as to suggest a shape to the rhythm. The music example that is the basis for this activity is in four-four time; that is, there are four beats per measure, and each beat is represented by a quarter note. It is not necessary for students to grasp the concept of measures for this activity. It can be explored in a discussion, but you would do better to wait until after the first problem so students have the chance to develop methods of counting beats on their own before they are given any instruction.

STEP 2 Problem 1: Determining the entry point for a rap phrase

TRACK 30

This problem is an example of a typical calculation a producer or an arranger makes when programming a sequenced production of a rap song or when cuing a singer to enter the recording. The part to be rapped is the seven-syllable lyric line "Now we want to take you there." The writer wants each syllable to fall exactly on consecutive beats with the last syllable occurring on the same beat on which the band enters, as heard on the CD track. The problem for students is to determine on which beat to start the rap so that this will be accomplished.

Introduce this problem with minimal instruction. The problem statements are on the resource page, The Producer's Problems. After students read through the problem description, play CD track 30.

Many students will try to solve the problem by feeling it out or guessing before resorting to calculations. Don't discourage this. When they get frustrated with guessing, they will move to counting and calculating. It is valuable for students to discover this necessity on their own.

Tell students you will play the CD track as many times as necessary. Encourage them to try to rap it correctly and to have fun with it. There will always be a few laughs and great opportunities for closet performers (and the not-so-closet performers) to get some attention. Students can put their scratch work in the space for Problem 1a on the worksheet.

You can expect to see students use many different strategies. Some will make tally marks to help them count the beats, while others will count mentally. Very few will use an algebraic equation on this simple problem. At this point, do not encourage any particular strategy.

Much of the music heard in the media and in popular formats is performed by computers using sequencing software and digitally sampled instrument sounds. The equipment for such productions uses a system called Musical Instrument Digital Interface, known as *MIDI*. When producers program music for MIDI, they often have to make calculations for the instrument and the vocal parts as well as the overall arrangements. Such calculations are similar to those used in solving the problems in this activity.

When the rap has been performed correctly, ask students to share how they solved the problem. Some students will probably be unable to perform the solution, but be sure that all students understand the solution and answer Problem 1b.

STEP 3 Problem 1c: An algebraic solution

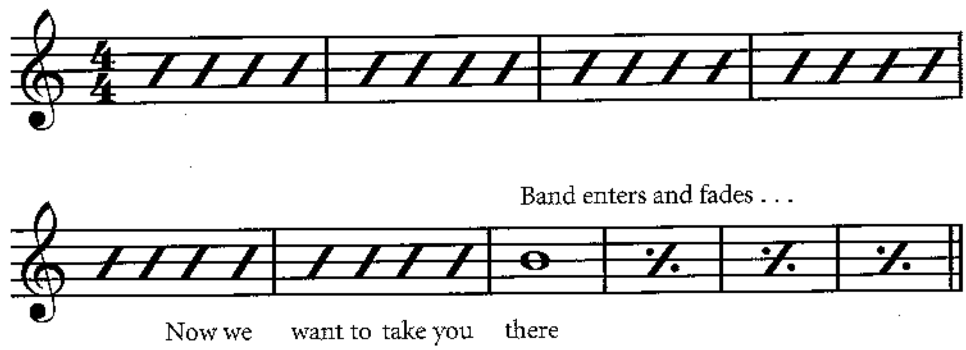
While most of your students will be able to do the arithmetic to solve the problem, not as many will be able to easily represent it as an algebraic equation. To help students see the connection, translate their written explanations to an algebraic sentence (equation). It is valuable to point out to students that they often apply algebraic concepts in mental mathematics without realizing it. This problem is a perfect example. Ideally, you can have a student put his or her explanation on the board, and you can then translate it to algebra. You can use the following analysis to demonstrate the solution:

$$\begin{array}{ccccc} \underbrace{\text{beats before rap}} & + & \underbrace{\text{beats of rap}} & = & \underbrace{\text{total beats}} \\ x & + & 7 & = & 25 \\ & & -7 & & -7 \\ & & & x = 18 & \end{array}$$

Most students will automatically perform the arithmetic on the right side of the equation, $25 - 7 = 18$, without using a variable. The same students can be mystified by the idea of subtracting 7 from both sides of an equation. The above analysis can show them the connection. This simple example does not make a strong case for using an equation with a variable because the arithmetic solution was obvious. Urge students to withhold judgment on the usefulness of equations—it will become more apparent when they encounter more difficult problems. Ultimately, try to bring students to see that the arithmetic solutions they use to solve the problem are in fact the steps used to solve the algebraic equation.

Shown on the following page is the standard musical notation used in marking rhythm. Notation of this sort is often referred to as a *chart.* (A chart in jazz and pop music refers to written musical parts for musicians that often contain chord symbols with rhythmic notation, combined with notated melodies.) It is similar to what you may see some of your students doing on their own (making tally marks). In general, music is written on five parallel lines called a *staff.* The lines and spaces of a staff correspond to various musical notes. In contexts like those in this activity, which involve rhythm but not notes, slash marks are used on the staff as shown. Each slash mark corresponds to a beat, and each vertical line (bar line) groups the beats into measures.

Use the blank Music/Rap Charts from the student worksheet on an overhead transparency as you analyze the beats with your class. The chart that follows shows how Problem 1 would be notated.



Students who made tally marks on their scratch paper will be excited to see that what they did on their own is very similar to how professional musicians write this type of music. Bring this to their attention.

You will need to decide whether or not to provide students with the Music/Rap Charts to use as worksheets. If students are having difficulty with the problems or have minimal experience with variables and equations, the charts may be necessary. But be aware that providing the charts will decrease students' reliance on using equations and calculations to solve the problems.

STEP 4 Problem 1d: Generalizing for rap length *r*

Allow students to work on this problem independently in groups. When most have come up with an answer, ask them to share their solutions.

Some students may be unclear as to what variable to solve for. The problem is purposely worded in terms of its application intent—to provide information on how to cue a singer to enter. Students need to make the connection that for this to happen they need to set up the formula so it yields the number of beats before the rap, x. If some groups wonder what variable to solve for, try to coach them through this reasoning rather than telling them that they need to express x in terms of r. If some groups have other difficulties with the generalization, point out that they can refer to the previous problem and simply use r instead of 7 and then solve for x.

An analysis to demonstrate the solution to students is shown below.

$$\underbrace{\text{beats before rap}}_{x} + \underbrace{\text{beats of rap}}_{r} = \underbrace{\text{total beats}}_{25}$$

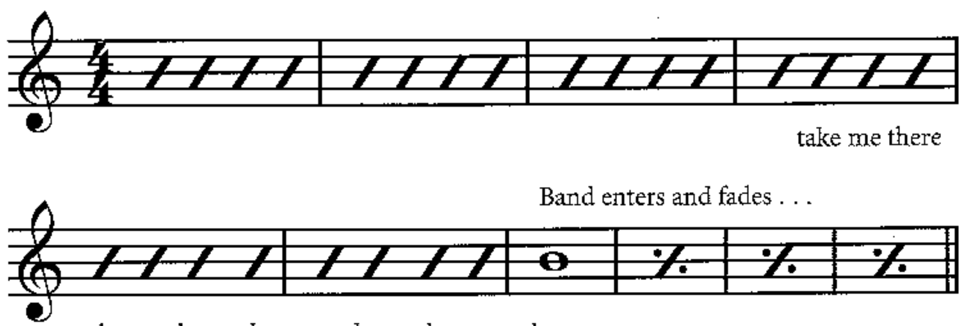
$$\begin{aligned} x + r &= 25 \\ -r \quad &\quad -r \\ x &= 25 - r \end{aligned}$$

STEP 5 Problem 2: Phrase made from repeating a rap

TRACK 30

The remaining problems follow a process similar to that outlined in Steps 2–4.

Direct students to read Problem 2 on the resource page. After they have read it, play CD track 30. When students are able to perform the part, have them work independently to finish the remaining parts of Problem 2 on their worksheets. Have students share their results with the class. A musical chart of the rap part, shown below, can be used to examine the problem and connect students' counting to the mathematical aspects.



As for Problem 1, you can use the following analysis to demonstrate how the arithmetic solutions students use to solve the problem are in fact the steps used to solve the algebraic equation representing the problem.

beats before pattern + beats of pattern = total beats

beats before pattern + (number of repetitions)(beats of rap) = total beats

$$x + (4)(3) = 25$$

$$x + 12 = 25$$

$$-12 \qquad -12$$

$$x = 13$$

STEP 6 Problem 3: Finding a phrase length

TRACK 30

Ask students to read Problem 3; then discuss it with the whole class so that all students understand what is being asked. Students will first need to calculate (no guessing) before they can try to perform, since the words are not written and they do not even know what to sing yet. They must calculate the length of a rap phrase (number of syllables) that, when repeated three times, will start with the bass guitar on the third beat of the third measure and end where the band enters. In light of this, the worksheet directs students to an algebraic solution as their first step. Play the CD track and encourage students to use a musical chart to aid them. The algebraic analysis appears on the top of the next page.

$$\text{beats before bass guitar} + \text{beats of pattern} = \text{total beats}$$

$$\text{beats before bass guitar} + (\text{number of repetitions})(\text{beats of rap}) = \text{total beats}$$

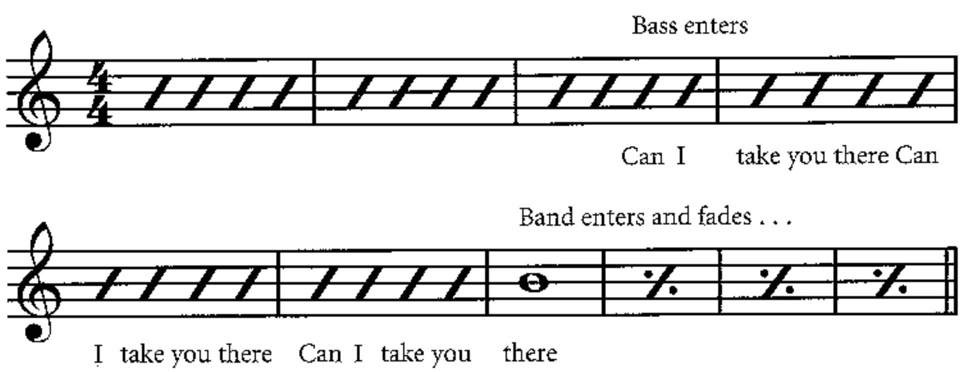
$$\begin{aligned} 10 + (3)(r) &= 25 \\ 10 + 3r &= 25 \\ -10 \qquad\quad & -10 \\ 3r &= 15 \\ r &= 5 \end{aligned}$$

Many students will count beats from where the bass enters to determine the total number of beats. This would simplify the solution. You might mention that, while this method is easier, counting from the beginning of the section would be necessary in programming computer-sequenced parts.

After students have calculated the number of syllables for the phrase, they need to write some words. You can have fun with this, but it can also take the class off on a tangent. You might suggest to students that they consider where the phrases overlap with the previous parts and write words so that some will be in sync. Two suggestions that honor this to varying degrees are "Can I take you there?" (three words overlap with the first phrase) and "Want to take you there" (all words overlap with the first phrase).

If you have time, you can have several groups of students try several versions of the rap. Don't allow this to distract from the mathematical focus of the activity. If you find the writing of words to be too much of a distraction, assign students the suggested phrases.

The musical chart below uses the phrase "Can I take you there?"



After students have performed the parts with the CD, let student groups finish the generalizations (Problem 3c–e) on their own. Before moving on to Problem 4, share solutions and analysis with the class, using a process similar to that outlined for the previous problems.

You might consider saving Problem 3e for a homework assignment, completing it at the end of the activity, or even skipping it altogether. If students' skill level with algebraic symbol manipulation is low, time spent on this problem can disrupt the momentum of the activity, and it is best saved for later. On the other hand, if students can solve the other problems with little difficulty, this problem can add richness and variety to the interactive exercises.

STEP 7 Problem 4 (optional): Using space between words

Problem 4 can be omitted or addressed during a subsequent class session, depending on how quickly your students have moved through the previous exercises.

This problem is different from the others in that the phrase considered contains some beats with no words. It is a four-beat phrase consisting of two words followed by two empty beats. The empty beats are referred to in music as *rests* (see Measures of Time, Part I). Since the requirements of the songwriter are that the end of the phrase must be a rapped word and the phrase as defined should end with two beats of rest, the five repetitions of the phrase must actually be four and a half repetitions, or five repetitions minus the last two beats of rest. This gives rise to various interpretations for an algebraic solution, and in any event a more complicated one. Your students will probably find it easier to work with the musical chart or with arithmetic to find the solution.

Play CD track 30 and have students rap their solutions. Following are an algebraic solution and a musical chart for you to use while discussing the problem with students.

$$\text{beats before pattern} + \text{beats of pattern} = \text{total beats}$$

$$\text{beats before pattern} + \overbrace{(\text{beats of phrase})(\text{repetitions}) - \text{rest beats}} = \text{total beats}$$

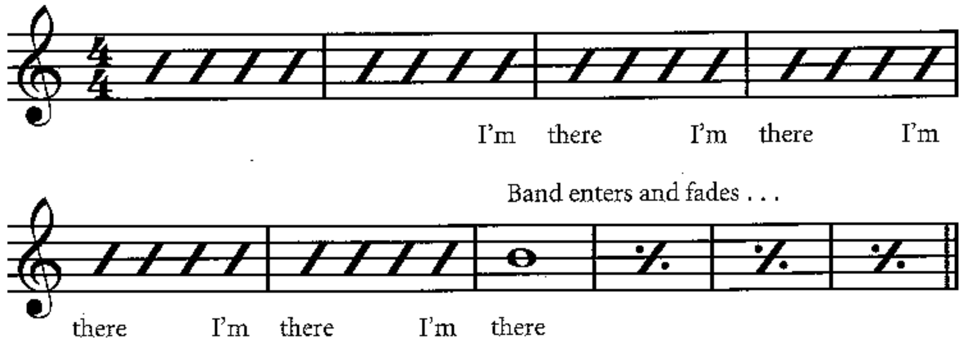
$$\underbrace{\text{beats before pattern}}_{x} + \overbrace{(\underbrace{\text{word beats}}_{2} + \underbrace{\text{rest beats}}_{2})}\underbrace{(\text{repetitions})}_{(5)} - \underbrace{\text{2 rest beats}}_{2} = \text{total beats}$$

$$x + (2 + 2)(5) - 2 = 25$$

$$x + 18 = 25$$

$$-18 \qquad -18$$

$$x = 7$$



STEP 8 No problem! Performance and analysis of the rap parts

Divide the class into four performance groups and assign each group a different rap part from Problems 1–4. Play the CD track and have students perform all the raps simultaneously. Experiment with performing different combinations of the parts together to find the combinations that are the most interesting or pleasing to the class. Most students will have to refer to their solutions and count in order to perform the parts successfully.

If you have time, hold a class discussion on the written analysis questions in the Follow-up Activities. If you do not have time for discussion and you plan to have students do a journal exercise or an extension, after the performance have them take notes on their observations of how the parts seemed to work together, making judgments on which combinations they liked best and why.

FOLLOW-UP ACTIVITIES

Written analysis

Have students use their discussion notes and problem solutions to respond in a written analysis to the following prompts:

- Which combinations of raps did you think sounded best together? Explain why.
- What could be changed in the other combinations to make them sound better? Explain why in terms of their mathematical relationships.
- Which of the formulas that you created could be most useful in helping you work out your proposed changes and perform them?

Writing prompts

Either instead of or in addition to the written analysis, it is valuable to have students reflect on their learning process and on some of the affective issues raised in the activity. The following prompts are suggested:

- What did you learn in this activity?
- What was the most interesting or valuable thing you learned?
- Was the activity fun?
- Did you realize before doing this activity that rock and rap music production involves algebra?

Homework assignments

Follow Record-Producer Algebra with practice solving equations. The often dry and tedious algorithms will have a fresh sense of relevance. Use exercises that involve creating algebraic models, solving linear equations, or solving general formulas for specified variables or word problems that emphasize writing equations.

Projects

- Make the changes you suggested in the written analysis and perform them for the class. Show how you used the formulas and which formulas are useless and why.
- Interview a real record producer and find out what kinds of mathematical problems producers actually face. Present your findings to the class.

ANSWERS

The Producer's Workstation

1b. Answers will vary. Example: "We counted all the beats that there were before the band came in and came up with 25 beats. Since the phrase has 7 syllables, we subtracted 7 from 25 and got 18. We knew we had to start the rap the beat after the eighteenth beat."

1c. $x + 7 = 25$
$x = 18$. The singer must enter on beat 19.

1d. $x + r = 25$
$x = 25 - r$

2b. Answers will vary. Example: "Since the phrase has 3 syllables and needs to be repeated 4 times, we multiplied 4 times 3 and subtracted that from 25. This gave 13, so we started rapping on beat 14."

2c. $x + 4(3) = 25$

$x = 13$

2d. $x + n(3) = 25$

$x = 25 - 3n$

3a. $10 + 3r = 25$

$r = 5$

3b. Answers will vary. Suggestion: "Where are we going?"

3c. $10 + nr = 25$

$r = \frac{15}{n}$

3d. $x + nr = 25$

$r = \frac{(25 - x)}{n}$

3e. $x = B - nr$ $\quad n = \frac{B - x}{r}$

$r = \frac{B - x}{n}$ $\quad B = x + nr$

4b. $x + (2 + 2)(5) - 2 = 25$

$x = 7$

4c. $x + (r + s)(5) - s = 25$

$x = 25 - 5r - 4s$

4d. $x = B - (r + s)n + s$ $\quad n = \frac{B + s - x}{r + s}$ $\quad r = \frac{B - x - s(n - 1)}{n}$

$B = x + (r + s)n - s$ $\quad s = \frac{B - x - rn}{n - 1}$