Introduction

Functional Melodies is intended to be taught as a supplement to a primary mathematics curriculum. Students use the activities to actively experience the integration of music and mathematics by listening to and performing music as they review mathematical concepts, solve problems, and perform calculations. For the most part, the activities are independent of each other, have no musical prerequisites, and can be used in any order.

Read through this introduction before starting the activities. It will explain the nature, history, and pedagogy of the music and mathematics connection and provide specific instructions for using this book. You will be best equipped to adapt and modify the activities to suit your needs and style if you understand the music and mathematics connection, as well as what outcomes to expect for student learning and classroom culture. Given that you are reading this book, you already are interested in its potential. Let your excitement show in the classroom. Your enthusiasm, confidence, and inspiration around the connections that the activities make can be as important for your students' success as any logistical preparation.

Care has been taken to make the materials friendly for all mathematics teachers regardless of their musical background. The audio tracks on the accompanying *Functional Melodies* CD allow you to conduct most of the activities without the use of musical instruments; all that you need is a CD player. There are, however, opportunities for you and your students to use musical instruments in place of the CD, turning your mathematics class into a music studio.

The Music and Mathematics Connection

The complex connection of music and mathematics exists on many levels, from the concrete to the abstract. The activities in *Functional Melodies* demonstrate the music and mathematics connection on four of these levels: the physics of sound, musical language, aesthetics, and metaphor.

Physics of sound Frequency, waves, resonance, vibration, and the mechanics of how musical instruments create pitch are all easily modeled by mathematics. Activities 10 and 11 (Scaling the Scale, Parts I and II) deal directly with the physics of sound.

Musical language The systematic organization of pitch and rhythm for the creation of music can be thought of as the mechanics of musical language. Pitch-interval relationships that create scales and chords, harmonic relationships, the division of time into patterns of rhythm, and musical form all contain elements that relate to each other by a set of quantitative rules with their own musical symbols. This connection is still at the mechanical level, and although it does imply rules of aesthetics and art, it is not concerned with that dimension. Activities 2 through 4 (Measures of Time, Parts I and II, and The Multiples of Drummers) deal with pitch and rhythm relationships.

Aesthetics Aesthetics is an intriguing level of analysis where the role of mathematics lies between the practical and the metaphorical. Mathematical analysis lends insight into aesthetically pleasing harmonic, melodic, and rhythmic patterns in Activity 6 (Functional Composer) and Activity 9 (Inside Out).

Metaphor The metaphorical connection between music and mathematics is largely based in process and analogy. The process of composing music can be likened to mathematical problem solving, and many musical systems and structures can be viewed as analogous to physical structures that can be modeled by mathematics. Activity 1 (Sound Shapes) is a direct example of this. Activity 9 (Inside Out) also treads in this area.

Connections Through History

For the Pythagoreans of ancient Greece (approximately 500 BC), the study of numbers and how they relate to musical harmony was considered the path to reaching spiritual understanding and purity of soul. Together, music and mathematics provided keys to the secrets of the world. The doctrine of the "music of the spheres" held that human souls must be attuned to the laws of the universe and suggested that planets in space produce sounds, a singing universe. (Later, Plato clarified this concept in his writings of *The Republic* and *Timaeus.*) The *Quadrivium,* the highest curriculum studied by the Pythagoreans, included music, geometry, arithmetic, and astronomy. Music was studied entirely as a mathematical medium. In fact, music was considered so purely mathematical that there is no record of its aesthetic dimension at this time, and there is virtually no record of folk music from early Greek culture.

Since the Pythagoreans, there have been many musician-mathematicians. Nicomachus (AD 100), Ptolemy (AD 165), Boethius (AD 500), Kepler (1600), Mersenne (1600), and Bernoulli (1700) all published works and led their own music and mathematics movements. Particularly fascinating is the work of Joseph Schillinger in 1945. The *Schillinger System of Musical Composition* was a massive undertaking employing a multitude of mathematical structures and formulas to aid in the refinement of musical composition by solving musical problems and communicating and classifying musical ideas. George Gershwin, Schillinger's best student, used this system to a significant extent in the opera *Porgy and Bess.*

The past several decades have seen a revolution in the way music is performed by many popular artists and used by the media in advertising, television, and the movies. The advent of digital sampling technology, computer sequencing, and MIDI (Musical Instrument Digital Interface) capabilities have often reduced the performance element of music to a process of computer programming and MIDI configurations. Many of the instruments heard in popular music and advertising are computer performances of digitally sampled sounds. Music is even being composed by computers. At Stanford University, the Center for Computer Research in Music and Acoustics (CCRMA) is pioneering research in the application of mathematical algorithms to musical composition.

Some may find the idea of the mathematization of music performance and composition a bit disturbing. Though today's high school students have grown up with computer-generated music, they often express pointed concerns about the mathematization of art and aesthetics. The activities of *Functional Melodies* can increase your students' awareness about the many ways mathematics can relate to aesthetics.

Art, Aesthetics, and the Domain of Mathematics

Why do we find a piece of music pleasing? What distinguishes random noise from music that captures our ear? How does a musical composition work? What holds the parts together? And, what does mathematics have to do with any of this?

These questions on the music and mathematics connection can lead to thought-provoking discussions—many of which are raised in the activities of *Functional Melodies.*

The creative process Musical composers draw from two primary domains in their work: inspiration and technique. Sometimes composers do not think of anything specific during the composition process; the music virtually appears to them, and all they have to do is write it down. This type of music comes from pure inspiration with no conscious application of theory or technique. On the other hand, when they lack inspiration, composers can use various formulaic methods to think through the creative process and compose their music. In practice, most artistic efforts are a blend of these two extremes. A bout of inspiration comes along and the composer's theoretical understanding and technique allow him or her to realize and articulate the message. The creative process swings between conscious application of discipline (rational, mathematical) and pure inspiration (intangible, spiritual). In the end, any aesthetically pleasing music contains some structure that binds it together and gives it a coherence that communicates. The search for this unifying structure of aesthetics leads directly to mathematics.

The role of mathematics Our discussion suggests that successful music contains a mathematically unifying structure. Is the converse true? Does the presence of mathematical integrity guarantee the success of a piece of music? Can the elusive artistic elements of inspiration and "chemistry" in music be measured or modeled by mathematics? Pythagoras asserted that numbers held the potential to explain all of reality. But as any experienced musician knows, a successful performance involves more than technical or mathematical correctness.

An intriguing idea to consider for classroom discussion is how the advent of the new mathematics of chaos and fractals might be able to probe the subtleties of music performance. Infinitely iterating models may be able to provide mathematical insights into realms commonly perceived as intangible. These can be potent topics for students. But be forewarned: I have had some students become very upset at the notion of using mathematics to analyze music on the emotional level. I have had to reassure them that there will always be domains beyond the mathematically measurable. Through these discussions, I have also discovered attitudes in my students that I never knew existed and I have gained some insights into hidden effects of mathematics schooling in general. A surprising number of students have very distinct and sometimes limited ideas about what mathematics can and should be used to explain. This is cause for wonder on the part of educators—to what extent might some mathematics curriculum and instruction stifle creativity and open-mindedness in our students?

Teaching and Learning with *Functional Melodies*

To use this book most effectively, it is important to consider some of the pedagogical premises on which it is based.

Mathematics as the story, music as the language The work of Howard Gardner on multiple intelligences suggests that for education to address the needs of all students, information must be made available to them in many forms. If a story needs to be told but is written in a language that the audience does not understand, the story must be translated to the appropriate language to be comprehended. In *Functional Melodies,* mathematics is the story and music is the language that makes it comprehensible. Students perceive and measure quantitative relationships (mathematics) through sound (pitch) and body interaction (rhythm). Many people can more readily hear the nature of a quantitative relationship or pattern than they can calculate it numerically or perceive it visually. In the two Functional Composer activities, the quantitative relationships of graph transformations are expressed aurally, visually, and numerically, so that the nature of the relationships are accessible to a broader audience, and can be understood more deeply at their mathematical level.

Depth of understanding through varied contexts How do we really get to know a concept—or our best friend, for that matter? Deep understanding is gained by experiencing the subject in a variety of contexts and viewing it from many perspectives. We get to know people better when we share experiences with them in unique situations, such as traveling to a foreign country. Getting to know mathematics can happen in a similar way, and doing mathematics in a musical context can reveal the essence of a mathematical principle from a unique, and otherwise unobtainable, perspective. In my personal experience with students and teachers, I often hear comments such as, "I never *really* understood functions until I saw this. . . . "

Balancing the classroom culture Much research has been done on the social construction of intelligence in classrooms and its effect on learning. Any experienced teacher has witnessed how a pecking order of "smart kids" versus "slow kids" can evolve between students. This can polarize the learning environment and program certain students to failure. Astute teachers aware of this seek out methods to establish a classroom culture that overcomes students' perceptions of ability. When a struggling mathematics student who is musically inclined suddenly becomes the expert in a class activity and is sought out for help by a "smart kid," an invaluable shift takes place in the classroom status culture. Students' confidence is enhanced and the learning environment is made safer.

Cooperative skill building Most of the activities in *Functional Melodies* require interdependence between a pair or group of students. This design of intentional interdependence maximizes student interaction. Students learn the efficiency of working with peers.

Enhancing appreciation for the scope of mathematics Interdisciplinary connections enhance a sense of meaning that motivates students to learn. When a subject such as mathematics is connected to an integral part of adolescents' culture such as music, mathematics is made fun and revealed to be a part of their worlds in a way they never imagined. While isolating disciplines can be instructionally efficient in many instances, it often leads to a sense of irrelevance and meaninglessness. Integrating disciplines reconstitutes the world, returning it to its authentic state—whole, interactive, and interdependent.

Learning Pathways

Learning takes place in many ways in heterogeneous classrooms, depending on the type of instruction practiced by teachers and the structure and content of curriculum. To effectively evaluate curriculum and instruction, it is important to realize that learning occurs both directly (immediately) and indirectly (delayed). In the case of direct learning, students acquire information, insights, and skills directly from the teacher or through curricular activities. The outcomes of direct learning are clear and assessable immediately following the instruction. Indirect learning, a more circuitous route of student growth, takes place as a more self-paced learning process within the student. With indirect learning, the ultimate outcome may not be predictable nor can it be immediately evaluated. Curricular activities that promote indirect learning inspire curiosity and imagination, supply a connection lending meaning to another topic, or provide a ground for future experiences to be more fully understood. Indirect learning is a part of the constructivist learning model, where curriculum and instruction facilitate students to construct their own meanings and secure a solid understanding of the material.

Functional Melodies activities may impact students both directly and indirectly. The flowchart below presents a model of how this can happen. Each bubble in the flowchart indicates a different outcome a student may experience directly from an activity. At this point an indirect learning process may take over, leading the student through the other outcomes in the flowchart, and ultimately to achieving mathematical skill and understanding.

Learning Pathways in *Functional Melodies*