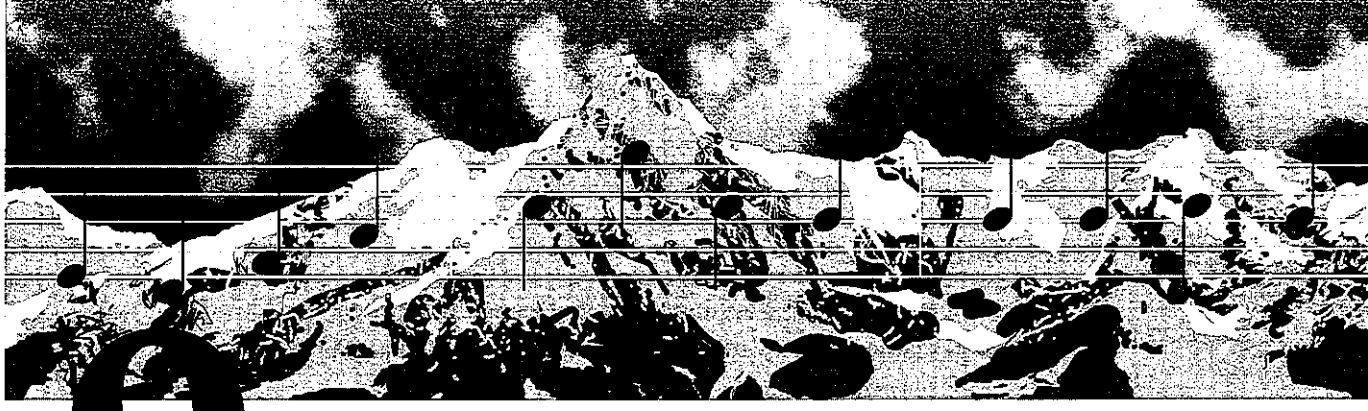


9

Inside Out

TRACKS 73–76

Hearing Pictures As Music Through Polar Coordinates

By using polar coordinates as an interface, Inside Out allows students to make music from graphic images. Students trace the outline of an image onto a polar coordinate system and determine the intersection points of the image and the graph at degrees 0, 10, 20, 30, and so on. They convert the polar distance of each point (r-value) to a corresponding musical pitch using the conversion key from Functional Composer. The notes correspond to the polar distances, while their order corresponds to the successive angle measurements. The result is a melodic line that embodies the shape of the graphic image. The melodies are performed by students; by listening to the melodies and viewing the original graphic images, the class matches each melody to the visual image that created it. The matching process becomes an exercise in which students must visualize how a ray with a constant angular velocity changes length over time. Students also create melodic lines from geometric figures and polar curves.

Since the students themselves create the melodies, at least one instrument and an instrumentalist who can read music will be needed for playing the unique melodies. You can illustrate the concept using the CD track for the fish example.

Mathematics topics

Polar coordinates and polar graphing, scaling of axes, angles, angle measurement, mathematics as an interface between two media. *Prerequisites:* Some experience with coordinate graphing, polar or rectangular.

Music topics

Musical notation, staffs, melodic contour, composition, imagery in music, instrumental performance. *Prerequisites:* At least one person who can read music to play the melodies.

Use with the primary curriculum

- To introduce polar coordinates in geometry and algebra 1
  While polar coordinates are not standard topics for geometry and algebra 1, introducing them with Inside Out can enhance students' ability to absorb the topic more fully in subsequent courses.
- To reinforce polar coordinate understanding in algebra 2
  When polar coordinates are studied as part of the primary curriculum, Inside Out can provide an intriguing application while reinforcing concepts.
- To show how mathematics can connect to the arts
  Used in any mathematics, music, or art class, Inside Out can provide a compelling example of how mathematics, images, and music are related.

Objectives

- To introduce the polar coordinate system
  Inside Out provides a fun and engaging context in which to learn about polar coordinates for the first time.
- To deepen understanding of the polar coordinate system
  The task of matching visual images to melodies created using a polar coordinate interface stretches students' ability to visualize the connection between polar curves and the changing angles and ray lengths that create them.
- To let students experience mathematics as a unifying tool and to stimulate their imagination
  Seeing how mathematics can be used as an interface between different media can motivate, inspire, and expand students' awareness of and appreciation for mathematics.

Student handouts

- From Sight to Sound (reading; one per student)
- Information Center (resource page; one per pair)
- Polar Coordinate/Musical Note Interface: 0°–180° (worksheet; one per pair)
- Polar Coordinate/Musical Note Interface: 190°–360° (worksheet; one per pair)
- Polar Coordinate Graph (worksheet; one per pair)
- Rectangular Coordinate/Musical Note Interface (worksheet for optional activity extension; one per pair)

Materials

- CD tracks 73–76
- Overhead transparencies of student worksheets
- Overhead transparency of Polar Coordinate Graph: Fish example
- Overhead transparency of Rectangular Coordinate/Musical Note Interface: Mountain example (optional activity extension)
- Musical instrument
- Assorted graphic material from magazines, catalogs, personal photographs, and so on (from you or students)

Instructional time

20–150 minutes

Instructional format

Inside Out follows a relatively open, project-style format. After the process and goal are made clear, students work in groups at their own pace to develop melodies from the graphic images. At the end, students come together as a whole class for performances, melody-image matching, and discussion.

Group students in threes so that the number of melodies to play will not be unwieldy. Most students will want their melody performed, and listening to and matching more than about ten melodies to corresponding images may become boring and tedious for some classes. Adding accompaniment and rhythm as described in Step 6 of the activity script will make the melodies more interesting and help keep students' attention for the performances of all the class's melodies.

You will need to identify musicians in the class who can perform the melodies and assist in providing rhythm and other embellishments. Designate these students as "musical experts" and distribute them as evenly as possible among the work groups.

Inside Out can take from one 20-minute session to three 50-minute class sessions depending on the approach used. Feel free to create your own version of the activity. Three possibilities are listed here:

- Short 20-minute presentation
  If you do not wish to invest a lot of time but want to expose your students to the ideas of the activity, present just the fish example.
- One or two class sessions
  You can provide graphic images that are compatible with Inside Out and have students spend one day learning the system, tracing images, creating melodies, and performing some of them. You will need at least part of another day for students to perform the balance of the melodies and discuss the outcomes.
- Two or three class sessions
  If students bring in their own graphic images (such as photographs, graphic designs, or logos), you will first need to introduce them to the graph of the fish example so they can see the size and style of image that will work for the activity. Tracing images, creating melodies, embellishing them with variations, performing them, and discussing results will fill two class sessions beyond the introduction day. Using their own images inspires students but requires a larger time investment.

Consider collaborating with an art or music teacher to make Inside Out a joint assignment. This will increase the importance of the project and also model how artists and mathematicians might work together to develop a common product.

Student preparation

Have students read From Sight to Sound. If students are providing the graphic images, you will have to do Steps 1 and 2 of the activity script several days prior to the rest of the activity.

ACTIVITY SCRIPT

Because students are working in groups and at their own pace, the resource page, Information Center, is longer and denser than the resource pages in other *Functional Melodies* activities. It includes most of the essential information students need in order to complete the activity. Find your own balance between how much you direct the steps of the activity script and how much you let students work though the activity on their own, guided by the resource page. Regardless of how much the students make use of the resource page, it can be a useful outline and reference for you.

An example

The example of a fish graphic and its melodic counterpart (CD track 73) are included primarily as a reference for you to see and hear how the activity plays out. Unless you plan to conduct a short version of the activity, do not demonstrate this example in its entirety with the students before the activity. Seeing and hearing a final product will dilute the element of mystery and discovery. You could play it as another example after the activity is completed or show students the fish graphic before the activity as an example of the kind of image that translates well.

The example uses only 180 degrees of the image, and some musical embellishment is added in the CD track. The bass line is created from the fish image and the higher string melody is a transformation of the fish melody using the function $y = -f(x) + 11$.

STEP 1 Discussing the reading and defining the activity

After students read From Sight to Sound, start a discussion with questions.

Ask students:
What points did the reading make?
What other kinds of interfaces do you know of?
What do you think about using mathematics to create music?
Are there ways that art conveys things that can't be understood using mathematics?

Give the students an overview of the activity by reading through the headings of the resource page, Information Center. They will need to have a good sense of what they are trying to accomplish in order to choose appropriate images.

STEP 2 Choosing a graphic image

If students bring in their own images, they need to be aware of some limitations. Graphics that cannot be reduced to a defined silhouette will not translate well to distinct coordinate values. Images that are essentially round with little variation will produce potentially boring melodies, possibly one repeated note. Student awareness of this will be spotty until after they have done the activity; having some images that produce boring melodies can be instructive as a point of discussion. A good way to equip the class to choose appropriate images is to provide examples of images that will not work well. On the other hand, the fish example is a good image because its contour is distinct, it has some variation in its form, and it fits comfortably on the coordinate graph.

Depending on students' understanding of the polar coordinate system and how it is used in this activity, their criteria for choosing images will vary. Some may focus on an image that they think will sound

good. Others will focus on images that they like for one reason or another or that have personal significance. Whatever their criteria, have them justify their choice thoughtfully.

Since you need to limit the number of images used if all the melodies are to be performed, each student group will need to agree on one image to use.

STEP 3 Tracing the images to the polar coordinate graph

If students find it difficult to see through the paper well enough to trace their image to the polar graph, they can cut out the image and use it as a stencil. Generally the center of the image should be near the pole of the graph, but students should feel free to experiment.

Ask students how the melody would change if the position of the image on the graph were shifted. This will be difficult for them to imagine before they do the activity, but it is valuable to float the question at this point and come back to it after the melodies have been played.

STEP 4 Determining the polar coordinates for the image

Each point at which the image intersects the rays emanating from the pole needs to be entered on the coordinate/note interface chart. The angles are already printed, so students simply need to count off the distance on each ray. This requires that students understand how to locate and represent points with polar coordinates and how to scale the polar graph for their image.

These and the other tasks are described on the resource page. Let students work through them using the resource page. Intervene with assistance or direct instruction only when students need help.

Scaling the graph will depend on the size of the image. Generally, if an image reaches to the outer edges of the graph, a scale of one note per unit distance from the pole will be appropriate. If an image is relatively small, a scaling of two notes per unit distance will make use of the full range of notes.

Explore with students the idea of scaling. Ask them how scaling will affect the resulting melody in general, and then ask what scaling would be appropriate for their image. Many will find it hard to make a judgment about this without having experienced the activity or polar coordinates. A scale of two or more notes per graph unit will create a melody with high contrast, and for many images the range might extend beyond the available notes. A scale of one half of a note or less per graph unit will create a melody with little contrast and limited range.

The Polar Coordinate/Musical Note Interface worksheet is in two parts to allow a student pair to divide tasks and work simultaneously while transcribing the points from a single graph. Point this out to the students and let them decide who will be responsible for each part of the graph.

STEP 5 Converting polar coordinates to musical notes

Once students have entered the polar coordinates on the coordinate/note interface chart, it is a straightforward process for them to use the number/note reference key to copy the notes that correspond to the polar distances onto the staff. Students do not need to understand the musical notation system to do this step. Some will be curious, and others will know how the notation works. Have the designated musical expert give a brief orientation, or give students the explanation in the following discussion point. In any event, students simply need to copy the notes they see from the key to the staff, paying attention to what line or space the note is on. They should do this in pencil to allow for modification in Step 6.

Musical notes are represented symbolically as dots on a set of five lines called a *staff*. They can be centered on a line or a space. Each line and each space of the staff represents a different note; notes high on the staff sound higher than notes low on the staff. Notes are given letter names, but musicians occasionally refer to them by number names as well. The rhythmic aspect of the notes uses another type of notation; see the activity Measures of Time.

STEP 6 Preparation for performance: Applying rhythm and accompaniment

The notes on the chart are essentially what modern musicians refer to as a *tone row*. These notes have some melodic content by virtue of the fact that they are a specific sequence of pitches, but as written they lack a key element of what creates a melody—rhythm. Students at this point need to decide how the notes will be played rhythmically and what kind of accompaniment will be used. How this step is handled will vary depending on both your musical inclination and that of your students. The activity can be successful even if no rhythm or accompaniment is applied and the notes are played exactly as they sit on the interface chart. Adding rhythm and accompaniment, however, can be fun for students and makes the latter stage of the activity more interesting for the class. Regardless of how far you and your students take this, some performance guidelines need to be adhered to for the melodies to be matched to their images. Go over the following guidelines with your class. Refer to Measures of Time for more information regarding these music basics.

9 Inside Out

- Each note must be assigned to a beat. The beat of a rhythm is an even pulse in time that all rhythmic events are related to. This assignment must be consistently adhered to by all groups or the melodies will not reflect the images accurately. Consider the fact that successive notes result from successive angles on the graph. If the notes are not consistently matched to beats in time, the resulting melody will imply a different placement of points on the graph and hence a different image.
- Choose a time signature and place bar lines on the staff to establish the rhythmic phrasing. It is okay to let the first few notes be a pickup. (A *pickup* is one or more notes that occur as an incomplete measure before the melody.) Where the first bar lines are placed can alter the rhythmic feel without changing the melodic shape in a way that would distort its representation of the image.
- Choose a designated background rhythm as an accompaniment for the performance. The three rhythmic accompaniments on the CD consist of a drum rhythm played in three styles: rock (track 74), Latin (track 75), and hip-hop/rap (track 76). Use one of these three, or ask a student to bring in a drum to accompany the instruments. The CD rhythms are all in four-four time.
- If notes on the interface chart repeat, you may choose to tie them to create a long note. This will add musical interest and shape to the melody while maintaining the representation of the image.
- Any type of dynamic shape or articulation can be applied as long as it does not distort the time value of the notes outside these guidelines. *Dynamics* in music is the relative volume of notes. *Articulation* refers to how specific notes and rhythms are played: staccato (short and clipped), legato (smooth and flowing), and combinations of these two. Experiment with your own combinations.
- Adding harmonic accompaniment (chords) from a guitar or piano can make the selections more musical and more fun to listen to, but don't let the accompaniment hide the melody. These chords would be a harmonious match for the notes used in the activity: C, Dmin, Emin, F, G7, Amin, and Bdim.

Students may be inclined to alter the melody rhythmically and embellish it to a point where it is not identifiable. To avoid squelching their creativity, suggest that they prepare two versions: one in which the melody is relatively pure so the class will have a good chance to match it to its image source, and another in which the melody is

altered as much as they like. More extensive alterations and compositions are suggested as extension projects.

STEP 7 The classroom concert and image matching

Assemble a core group of willing musicians as a class band or select an individual to play the melodies created.

The graphic images need to be displayed for the class to view as they listen to the melodies. Students can copy their images on the chalkboard, draw them on chart paper, or trace them on overhead transparencies. Making enlarged versions for the chalkboard or chart paper can be problematic if the resulting images are not very accurate and thus no longer correspond to the melody. The advantage of using the chalkboard is that most of the images can be viewed simultaneously and compared. Tracing on overheads allows for much greater accuracy, but flipping through ten or so slides for each melody to try to find a match can be tedious.

When a melody is played and students are trying to match it to the images, coach them to visualize the melody on the polar coordinate graph. Imagine a rotating ray around the pole to be the spoke on a bicycle wheel that moves 10 degrees with each beat of the music. Visualize the length of the ray changing as the pitch changes.

Ask students:
What part of the polar coordinates reflects time? [The angle.]
What part reflects the pitch of the note? [The polar distance.]

If students have difficulty, it can be helpful to talk through some of the examples in Step 8 to stimulate their visualization skills with familiar figures.

STEP 8 Discussion and exercises

The visualization exercises listed here can be done before the performances to enhance students' visualization skills with the interface, or they can be done afterward to assess how students have assimilated the ideas. It can be fun to have students attempt to sing the geometric figures. Consider presenting these exercises in the reverse application, giving students the sound and having them describe the geometric figure.

- What would be the sound of a circle with its center at the pole? [One long note, not changing in pitch.]
- How would the circle sound if its center were moved from the pole? [A smooth rise and fall in pitch over time.]

(continued)

- What would be the sound of a spiral starting at the pole? [A steadily increasing pitch, like playing a scale.]
- What would a square sound like? [Steady rise and fall in pitch.]
- What would a gear shape sound like? [Alternating pitches.]
- What would a star sound like? [Several scale steps up, then back, repeating.]

FOLLOW-UP ACTIVITIES

Textbook assignments

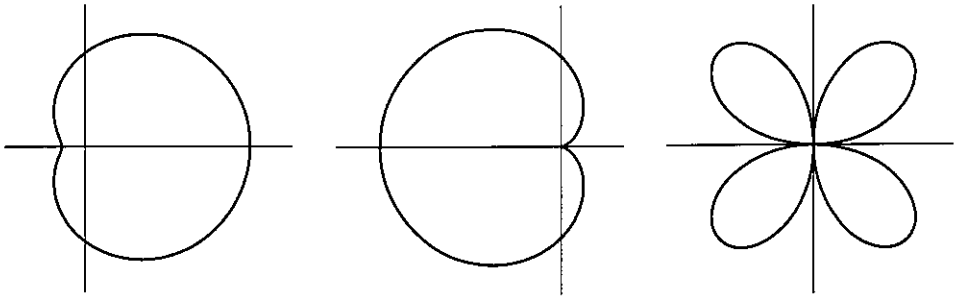
Follow Inside Out with textbook assignments that you would ordinarily use to follow an introduction to the polar coordinate system.

Writing prompts

- Did some images make better melodies than others? Which ones? Why?
- What types of images would translate well to images in this system?
- Did the mathematical system of polar coordinates work well to convey the image? Could music be written in another way that would reflect the image better? If so, how?
- Did you enjoy the activity? What did you learn?

Extensions

- Make music from shapes generated by standard polar equations, such as:



a. limaçon b. cardioid c. four-petal rose

- Make images from popular well-known melodies using the system of Inside Out, and let others attempt to determine the melody from the design.
- Use a Cartesian coordinate system to convert images to melodies. Included with this activity is a graph/note interface that can be used to convert images to melodic lines using a rectangular coordinate system. You can present this to students as a creative problem-solving project. Students will need to determine what scale to use for the axes, where to place the axes, which intersection points to use to best convey the image, and what note duration to assign to the horizontal axis units. An example of a mountain range converted to notes is supplied for reference. This example is fairly complex. Simpler versions can be done using only quarter notes. The curve created on the graph can then be represented mathematically as equations of lines or other curves over designated domain intervals. Students can compare the musical lines and mathematical representations in a class presentation.

Project

Using a melody created from an image, students can create variations using techniques from Functional Composer and write a musical composition. When students perform it for the class, ask them to explain why they chose the transformations they used and why those transformations sound good in terms of the mathematical relationships.