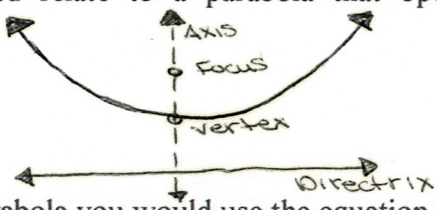
Parabolic Solar Collector

Introduction

The problem to be investigated in this project is to see which equation for a parabolic dish is the best in maximizing the temperature at the focus point. Another investigation includes seeing if any other areas around the main focus point achieve great amounts of heat. In order to answer these questions background information on parabolas, the formula to find the focal distance, light reflection, and the angle of incidence are needed.

The definition for a parabola is the set of all points equidistant from a fixed line and a fixed point not on that line. The line is called the directrix, and the point is the focus. Since parabolic shapes are based on the shape of a parabola, knowing how a parabola is created can be of great help. To help form a parabola the equation needed is $\mathbf{y-k = a(x-h)^2}$. The opposites of **h** and **k** equal the vertex because the x-coordinate in the vertex equals **h**, and the y-coordinate equals **k**. In this case **a** is equal to the direction the parabola opens. If **a** is less that zero the parabola opens downward. If **a** is greater the parabola opens upward. To find the other points on the parabola, plug in numbers to see if they fit the equation. To find the zeroes, or the set of coordinates were the **y** is equal to zero, make y=0 in the equation and solve for **x**.

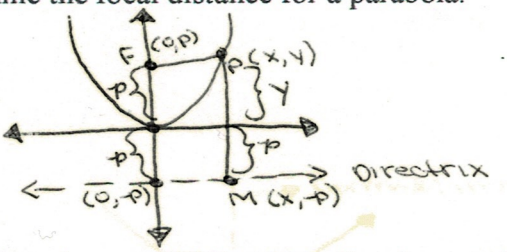
The relationship between the focus, directrix, vertex, and axis of symmetry on a parabola is not as complicated as it may seem. It is best to look at these relationships in this manner. The focus point and the directrix line are equal distance from the vertex. One is above and one is below the vertex. The vertex is the center of the parabola. The axis of symmetry is the line of the x-coordinate in the vertex, and the directrix is the line of the y-coordinate in the vertex. The focus point and the vertex lie on the axis of symmetry. These relationships stated relate to a parabola that opens upward and downward.



To find the focus point on a parabola you would use the equation **p=1/4a**, where **p** equals the focus and **a** equals the coefficient that comes from the equation of a parabola

and decides the direction of the parabola. The formula could also be changed to **a=1/4p** if you were trying to find the **a** of an equation. To understand how this equation was derived it is best to see it in the following way.

*Find an equation to determine the focal distance for a parabola.



NOT DRAWN TO SCALE

According to the Locus definition there is a special line that all points on the parabola are equal to, therefore in this problem above, **FP=PM**. This line is called the directrix. To solve the above problem, which will help with the understanding of how the equation for the focal distance was derived, use the distance formula, $\mathbf{d} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$, to express the distance between **F** and **P**. The coordinates for **F** are **(o,p)** and the coordinates for **P** are **(x,y)**. Using the distance formula, the distance between **F** and **P** is $\sqrt{(x-o)^2 + (y-p)^2}$. With this it can be said that $\sqrt{(x-o)^2 + (y-p)^2} = PM$. Since the coordinate for **P** is **(x,y)** it means the length from the x-axis to that point is equal to **y.** Point **M** has a length of **p** because it is the same length as **F** to the vertex whose coordinates are **(o,p)**. This is so because the directrix is equal distance to the center of the parabola as the focus point, which is **F** in the diagram. Therefore the length of **PM = y + p** so $\sqrt{(x-o)^2 + (y-p)^2} = y + p$. This equation should be put into the parabolic form $y = ax^2$.

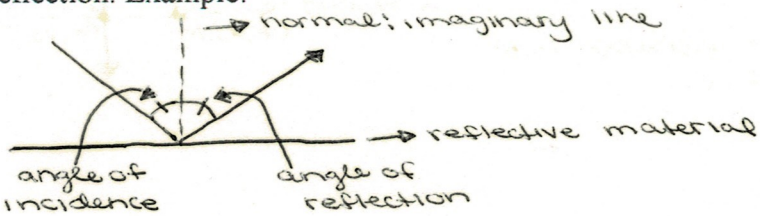
$$\sqrt{(x-0)^2+(y-p)^2} = y+p$$
$$(y-p)^2 + x^2 = (y+p)^2$$
$$(y-p)(y-p)+x^2 = y^2+2py+p^2$$
$$y^2-2py+p^2+x^2 = y^2+2py+p^2$$
$$-y^2+2py-p^2 \quad -y^2+2py-p^2$$
$$x^2 = \cancel{y^2}+2py+\cancel{p^2}-\cancel{y^2}+2py-\cancel{p^2}$$
$$\frac{x^2}{4p} = \frac{\cancel{4}py}{\cancel{4}p} \rightarrow \boxed{y = \frac{1}{4}px^2}$$

With this equation, **a = 1/4p** and **p = 1/4a**, which is the equation used to find the focal distance. This focal distance is the distance from the vertex to the point, on the axis of symmetry, which becomes the focus of a parabola.

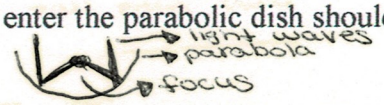
To complete this project, information about light reflection and the angle of incidence is needed. Light is energy that comes from vibrating electrons and travels in a straight line or by a transverse wave. The average speed at which it travels in air is

3.0×10^8m/s. To get a reflection a wave must reach a smooth surface and "bounce back." The smoother and greater the elasticity, or ability to retain its shape when you hit it, the better the reflection. The less smooth or elastic the less the reflection it has. For example, water has a reflection, but it is not as clear to see as a mirror because the mirror has more elastic.

The angle of incidence is related to the Law of Reflection. The law states that the angle of incidence equals the angle of reflection. Example:



The normal in the diagram is an imaginary line that can be used to determine the equality between the angle of incidence and the angle of reflection. This relates to the project because the parallel light waves that enter the parabolic dish should all reflect to the same point is called the focus.



Once again, the problem to be investigated includes finding the most effective equation for a parabola to maximize the temperature at the focus of the solar collector, with the origin being at **(0,0)**. Also to be looked at is the temperatures of spots around the focus. For this project a focus point from one to eight was to be chosen, and then the equation to create a parabola based on that point found. The hypothesis for the experiment goes as follow: It is believed that the most effective equation for a parabola to maximize the temperature at the focus of the collector is $\mathbf{y = 1/16x^2}$ where the focal point is four inches. This is so because the shape is in a medium state which may allow the light waves to be evenly distributed through the dish receiving the maximum temperature at the focal point.

Materials

Corrugated Cardboard (about four 3x3 foot pieces)
A role of aluminum foil
Scissors
Graph paper with 1x1 inch squares (one large sheet)
Pencil
Masking tape (one role)